RELATION BETWEEN DIMENSIONLESS NUMBERS AND DRYING TEMPERATURE COEFFICIENT

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The results of an analytical calculation of the relation between dimensionless numbers and drying temperature coefficient are presented.

The system of differential equations of heat and mass transfer for convective drying is solved for boundary conditions of the third kind. In this case the boundary conditions relate the values of the transfer potentials on the surface of the body with the corresponding potentials of the medium in terms of the laws of convective heat and mass transfer on the surface. Newton's law is usually used as the law of convective heat transfer and Dalton's law as the law of surface mass transfer.

The coefficients of heat transfer α_q and mass transfer α_m figuring in Newton's and Dalton's laws depend in the general case on the regime parameters and are variables for the period of a decreasing rate of drying.

Usually to simplify the solutions one assumes the transfer coefficients and temperature of the medium to be constant for the entire surface of the body. The system of differential equations of heat and moisture transfer in generalized variables for the one-dimensional problem has the form [1-3]

$$\frac{\partial T(X,\ \mathrm{Fo})}{\partial \mathrm{Fo}} = (1+\varepsilon\,\mathrm{Ko}\,\mathrm{Pn}\,\mathrm{Lu})\left[\frac{\partial^2 T(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X}\,\frac{\partial T(X,\ \mathrm{Fo})}{\partial X}\right] - \varepsilon\,\mathrm{Ko}\,\mathrm{Lu}\left[\frac{\partial^2 U(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X}\,\frac{\partial U(X,\ \mathrm{Fo})}{\partial X}\right]; \quad (1)$$

$$\frac{\partial U(X,\ \mathrm{Fo})}{\partial \mathrm{Fo}} = \mathrm{Lu}\left[\frac{\partial^2 U(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X}\,\frac{\partial U(X,\ \mathrm{Fo})}{\partial X}\right] - \mathrm{Lu}\,\mathrm{Pn}\left[\frac{\partial^2 T(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X}\,\frac{\partial T(X,\ \mathrm{Fo})}{\partial X}\right]. \quad (2)$$

The dimensionless boundary conditions of the third kind as applied to the system of Eqs. (1) and (2) can be written so:

$$\frac{\partial T(1,\ \mathrm{Fo})}{\partial X} - \mathrm{Bi}_q[1 - T(1,\ \mathrm{Fo})] + (1-\varepsilon)\,\mathrm{Ko}\,\mathrm{Lu}\,\mathrm{Ki}_m(\mathrm{Fo}) = 0; \quad (3)$$

$$-\frac{\partial U(1,\ \mathrm{Fo})}{\partial X} + \mathrm{Pn}\,\frac{\partial T(1,\ \mathrm{Fo})}{\partial X} + \mathrm{Ki}_m(\mathrm{Fo}) = 0. \quad (4)$$

The most typical form of the relation of the dimensionless flux of material for convective heat and mass transfer (2) is the relation

$$\mathrm{Ki}_m = \mathrm{Bi}_m[1 - U(1,\ \mathrm{Fo})], \quad (5)$$

i.e., the flux of material on the surface of the body is a function of the mass transfer potential. The problem is considered symmetric, i.e.,

$$\frac{\partial T(0,\ \mathrm{Fo})}{\partial X} = \frac{\partial U(0,\ \mathrm{Fo})}{\partial X} = 0, \qquad \begin{matrix} T(0,\ \mathrm{Fo}) \neq \infty, \\ U(0,\ \mathrm{Fo}) \neq \infty. \end{matrix} \quad (6)$$

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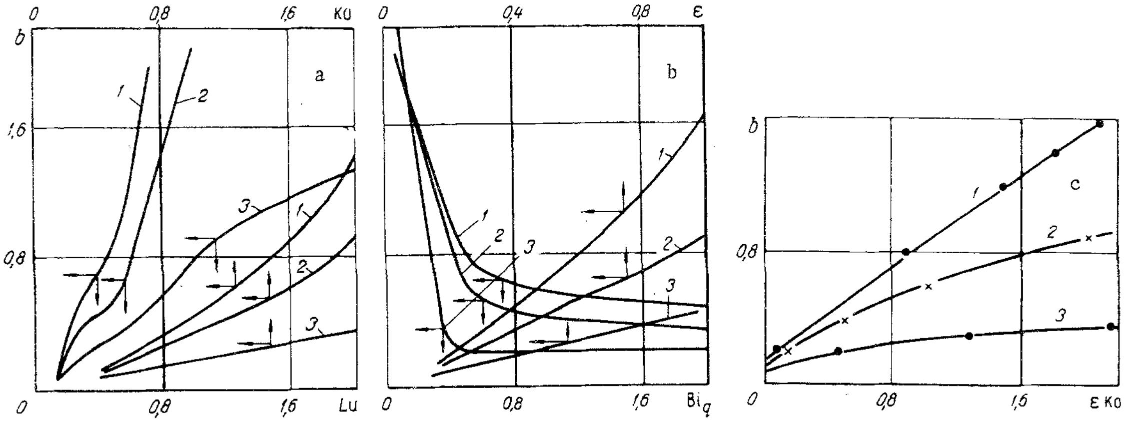

Fig. 1. Drying temperature coefficient as a function of the: a) Lu and Kossovich numbers; b) phase transition number and heat-transfer Biot number; c) product εKo; b = (dT/dFo)/du/dFo; 1) Fo = 0.5; 2) 1.0; 3) 10.

At the end of the period of a constant drying rate the distribution of $\bar{T}$ and $\bar{U}$ follows the parabolic law. However, we can use simpler initial conditions in the form

$$T(X,\ \text{Fo}) = U(X,\ \text{Fo}) = 0 \tag{7}$$

and the time is reckoned from the start of drying.

The solution of the system of equations of heat and mass transfer (1) and (2) with boundary conditions (3), (4) for an unbounded plate has the form [1]

$$T(X,\ \text{Fo}) = 1 - \sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}\cos(\nu_i\mu_n X)\exp(-\mu_n^2\,\text{Fo}); \tag{8}$$

$$U(X,\ \text{Fo}) = 1 + \frac{1}{\varepsilon\,\text{Ko}}\sum_{n=i}^{\infty}\sum_{i=1}^{2} C_{ni}(1-\nu_i^2)\cos(\nu_i\mu_n X)\exp(-\mu_n^2\,\text{Fo}). \tag{9}$$

The constant coefficients C_{ni} are determined by the corresponding equations [2]; they depend on the numbers ε, Ko, Bi_q, Bi_m, and Lu. In this case the solutions (8) and (9) have an approximate character in the sense that the laws of convective mass and heat transfer are inapplicable to the drying process in the period of a decreasing rate (2), (4).

The dimensionless rate of change of the potentials of transfer of heat and material can be obtained by differentiation of Eqs. (8) and (9) with respect to Fo:

$$\frac{d\bar{T}}{d\,\text{Fo}} = \sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}\,\frac{\mu_n}{\nu_i}\sin\nu_i\mu_n\exp(-\mu_n^2\,\text{Fo}); \tag{10}$$

$$\frac{d\bar{u}}{d\,\text{Fo}} = -\frac{1}{\varepsilon\,\text{Ko}}\sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}\,\frac{(1-\nu_i^2)\,\mu_n}{\nu_i}\sin\nu_i\mu_n\exp(-\mu_n^2\,\text{Fo}). \tag{11}$$

The drying temperature coefficient is determined from Eqs. (10) and (11)

$$b = \frac{d\bar{T}/d\,\text{Fo}}{d\bar{u}/d\,\text{Fo}}.$$

The solution of the system of differential equations of heat and moisture transfer gives the drying temperature coefficient as a function of a large group of heat- and mass-transfer dimensionless numbers

$$b = f(\text{Fo},\ \text{Lu},\ \text{Bi}_q,\ \text{Ko},\ \text{Bi}_m,\ \varepsilon).$$

However, not all dimensionless numbers equally affect the course of the process.

As a result of solving system of Eqs. (10) and (11) on a Minsk-22 computer, we obtained the results of the dependence of individual dimensionless numbers of heat and mass transfer on the drying temperature coefficient.

The Lu number has the maximum effect on the drying temperature coefficient. We see in Fig. 1a that with an increase of Lu the drying temperature coefficient increases. The most intense increase of coefficient b occurs at the smallest values of the Fo number. This is explained by the fact that at the start of the drying process the rate of change of temperature in the material outstrips the rate of propagation of the mass of the bound substance.

The surface heat- and mass-transfer Biot numbers Bi_q and Bi_m have a considerable effect on heat and mass transfer. Figure 1b shows the drying temperature coefficient b as a function of Bi_q. An analysis shows that for small values of Bi_q the rate of change of the heat transfer potential (dT/dFo) outstrips considerably the rate of change of the potential of the material (du/dFo), i.e., the drying temperature coefficient increases with a decrease of Bi_q. With an increase of Bi_q the transfer process intensifies, and simultaneously with this the gradients of the transfer potentials increase in the material. The dimensionless rates of change of the potentials of heat and material change such that with an increase of Fo the Bi_q number has an ever lesser effect on b. When Fo = 10 (Fig. 1b) the drying temperature coefficient b (for $Bi_q > 0.4$) does not depend on Bi_q.

The phase transition number ε has the same effect on the process of heat and mass transfer as the Kossovich number. The difference consists in the intensity of the effect on heat transfer: the effect of ε is considerably less than that of Ko. The graph of the effect of ε on b is analogous to the graph for Ko (Fig. 1b). With an increase of Ko and ε the temperature coefficient b increases. With an increase of the Fo number the effect of the Ko and ε numbers on b decreases, and when **Fo** = 10 the dependence becomes linear.

In conclusion we should note that the determination of the dependence of the drying temperature coefficient b on dimensionless numbers of heat and mass transfer permits establishing the relation between the Rebinder number and the transfer properties of moist material. For this purpose it is necessary to use analytical solutions of the system of differential equations of heat and moisture transfer in capillary-porous bodies.

The product εKo is often found in analytical solutions of the problem. We have presented the dependence of this complex on the drying temperature coefficient in Fig. 1c. The character of these curves, naturally, is analogous to the curves ε = f(b) and Ko = f(b).

NOTATIONS

$X = X/R$	is the dimensionless coordinate;
$T = (t-t^*)/t_c-t^*)$	is the dimensionless temperature;
$(u^*-u)/(u^*-u_p) = U$	is the dimensionless moisture content;
Γ	is the constant coefficient (for a plate $\Gamma = 0$);
α_q, α_m	are the heat and mass transfer coefficients,
μ_n,	are the roots of the characteristic equation;
ν_i	are the characteristic numbers;
ε	is the phase transition number;
Ko	is the Kossovich number;
Pn	is the Posnov number;
Lu	is the Luikov (relative inertia) number;
Fo	is the Fourier number;
Ki_m	is the Kirpichev number;
Bi_q	is the heat-transfer Biot number;
Bi_m	is the mass-transfer Biot number.

LITERATURE CITED

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